

CS 453/698: Software and Systems Security

Module: Bug Finding Tools and Practices

Lecture: Symbolic execution

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Spring 2025

Outline

- 1 Introduction
- 2 Conventional symbolic execution
- 3 Symbolic loop unrolling
- 4 Concolic execution and hybrid fuzzing
- 5 Weakest precondition

Illustration

```
1 fn foo(x: u64): u64 {  
2   if (x * 3 == 42) {  
3     some_hidden_bug();  
4   }  
5   if (x * 5 == 42) {  
6     some_hidden_bug();  
7   }  
8   return 2 * x;  
9 }
```

Illustration

Unit Test

foo(0);

foo(1);

```
1 fn foo(x: u64): u64 {  
2   if (x * 3 == 42) {  
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Illustration

Unit Test

foo(0);

foo(1);

Fuzzing

foo(0);

foo(1);

foo(12);

foo(78);

.....

foo(9,223,372,036,854,775,808);

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1 fn foo(x: u64): u64 {  
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Illustration

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Unit Test

foo(0);

foo(1);

Fuzzing

foo(0);

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.....

foo(9,223,372,036,854,775,808);

Symbolic execution

foo(x)

aborts when $x = 14$

returns $2x$ otherwise

Satisfiability Modulo Theories (SMT)

Definition: A procedure that decides whether a **mathematical formula** is **satisfiable**.

Example:

- $3x = 42$
- $2x \geq 2^{64}$
- $5x = 42$

Satisfiability Modulo Theories (SMT)

Definition: A procedure that decides whether a **mathematical formula** is **satisfiable**.

Example:

- $3x = 42 \longrightarrow$ satisfiable with $x = 14$
- $2x \geq 2^{64} \longrightarrow$ satisfiable with $x \geq 2^{63}$
- $5x = 42 \longrightarrow$ unsatisfiable, cannot find an x

Ask two question whenever you see a symbolic execution work:

- How does it convert code into mathematical formula?
- What does it try to solve for?

Program Modeling Desiderata

- Control-flow graph exploration
- Loop handling
- Memory modeling
- Concurrency

Outline

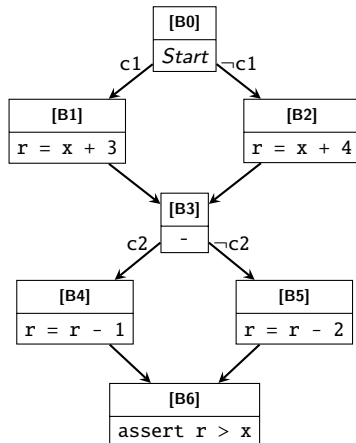
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An example of a pure function

```
1 fn foo(  
2   c1: bool, c2: bool,  
3   x: u64  
4 ) -> u64 {  
5   let r = if (c1) {  
6     x + 3  
7   } else {  
8     x + 4  
9   };  
10  
11   let r = if (c2) {  
12     r - 1  
13   } else {  
14     r - 2  
15   };  
16  
17   r  
18 }  
19 spec foo {  
20   ensures r > x;  
21 }
```

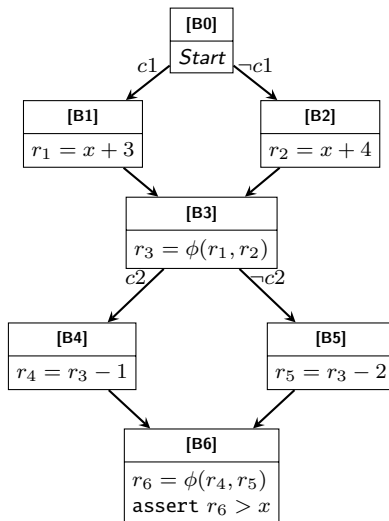
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```



The example in SSA form

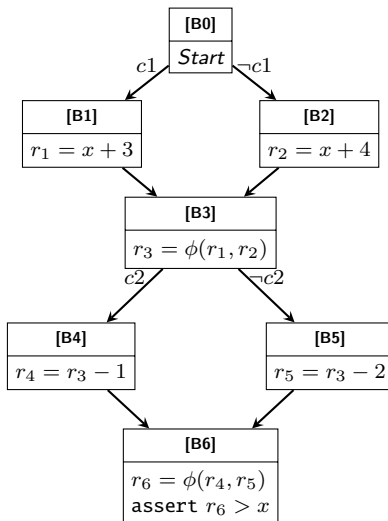
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```



Path-based exploration

Vars: $c1, c2, x, r_1-6$

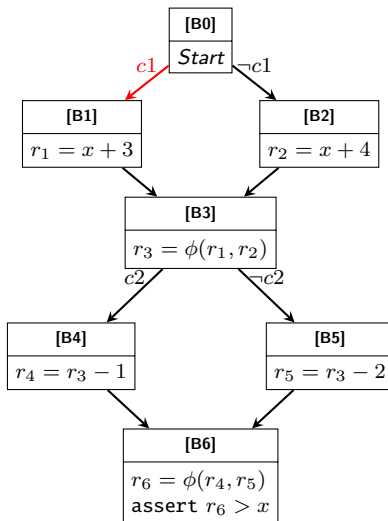
B0	Sym. repr. Path cond.	$\emptyset$ True
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Path-based exploration

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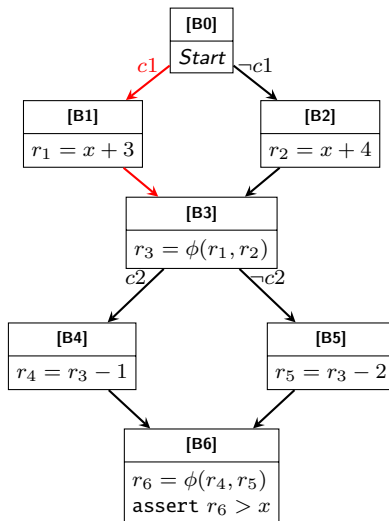
B0	Sym. repr. Path cond.	$\emptyset$ True
B1	Sym. repr. Path cond.	$r_1 = x + 3$ $c1$



Path-based exploration

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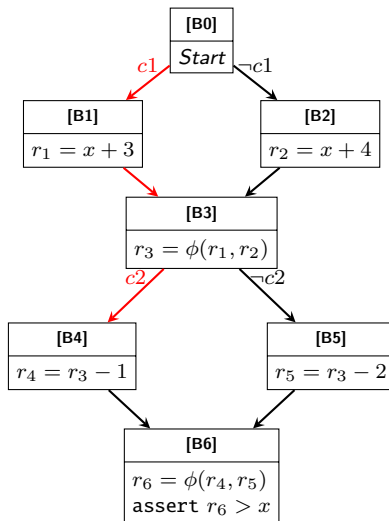
B0	Sym. repr. Path cond.	$\emptyset$ True
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Path-based exploration

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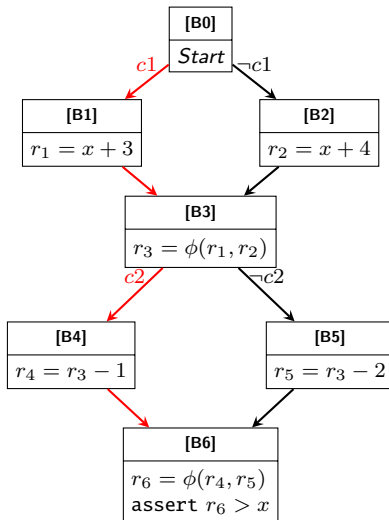
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B4	Sym. repr. Path cond.	$r_1 = x + 3$ $r_3 = r_1$ $r_4 = r_3 - 1$ $c1 \wedge c2$



Path-based exploration

Vars: $c1, c2, x, r_1-6$

B0	Sym. repr. Path cond.	$\emptyset$ True
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B6	Sym. repr. Path cond.	$r_1 = x + 3$ $r_3 = r_1$ $r_4 = r_3 - 1$ $r_6 = r_4$ $c1 \wedge c2$

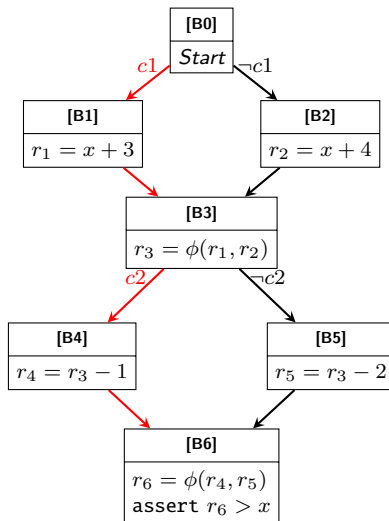


Proving procedure (per path)

Vars: $c1, c2, x, r_{1-6}$

B6	Sym. repr.	$r_1 = x + 3$ $r_3 = r_1$ $r_4 = r_3 - 1$ $r_6 = r_4$
	Path cond.	$c_1 \wedge c_2$

$\rightsquigarrow$



Proving procedure (per path)

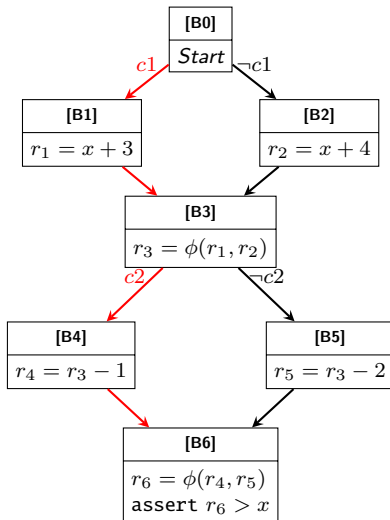
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	Path cond.	$c_1 \wedge c_2$

$\rightsquigarrow$

Prove that $\forall c1, c2, x, r_{1-6}$:

$((c1 \wedge c2) \wedge$
 $(r_1 = x + 3)$
 $(r_3 = r_1)$
 $(r_4 = r_3 - 1)$
 $(r_6 = r_4)$
 $)) \Rightarrow (r_6 > x)$

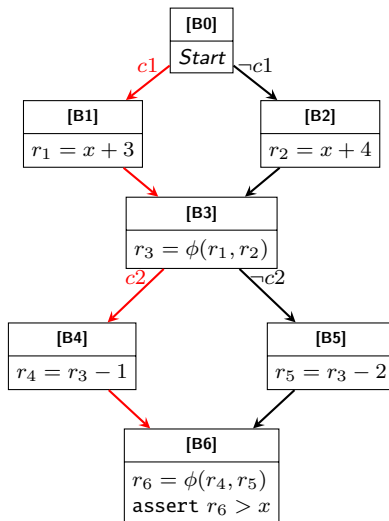


Proving procedure (all paths)

Prove that

$\forall c1, c2, x, r_{1-6}:$

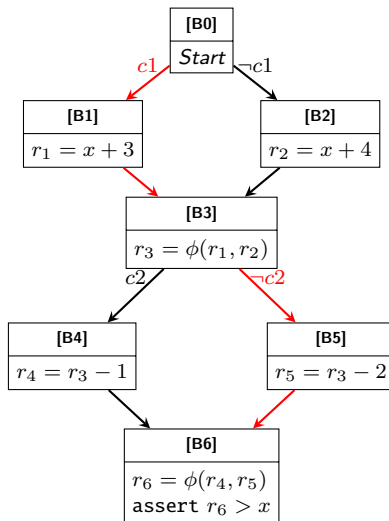
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 $(r_1 = x + 3)$
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Proving procedure (all paths)

Prove that

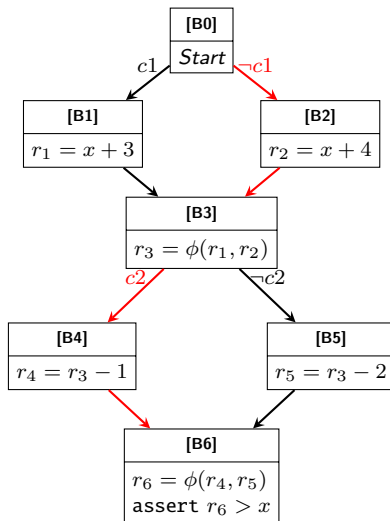
$\forall c1, c2, x, r_{1-6}:$

$$\begin{aligned} & ((c1 \wedge \neg c2) \wedge (\\ & \quad (r_1 = x + 3) \\ & \quad (r_3 = r_1) \\ & \quad (r_5 = r_3 - 2) \\ & \quad (r_6 = r_5) \\ &)) \Rightarrow (r_6 > x) \end{aligned}$$


Proving procedure (all paths)

Prove that

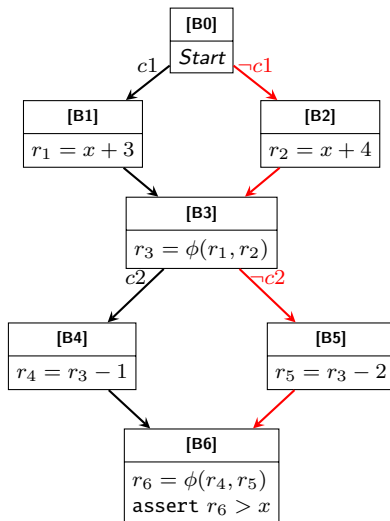
$\forall c1, c2, x, r_{1-6}$:

$$\begin{aligned} & ((\neg c1 \wedge c2) \wedge (\\ & \quad (r_2 = x + 4) \\ & \quad (r_3 = r_2) \\ & \quad (r_4 = r_3 - 1) \\ & \quad (r_6 = r_4) \\ &)) \Rightarrow (r_6 > x) \end{aligned}$$


Proving procedure (all paths)

Prove that

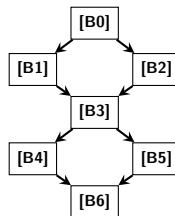
$\forall c1, c2, x, r_{1-6}$:

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Path explosion

Path explosion

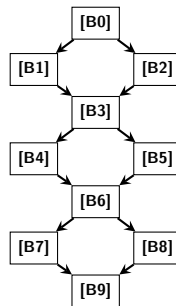
2^2 paths



Path explosion

2^2 paths

2^3 paths



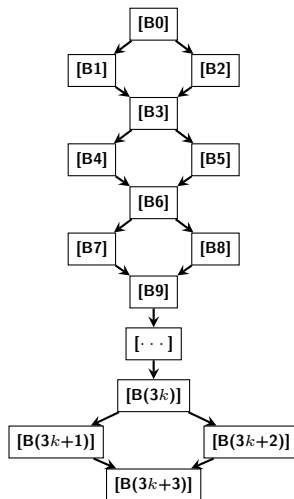
Path explosion

2^2 paths

2^3 paths

...

2^k paths



Outline

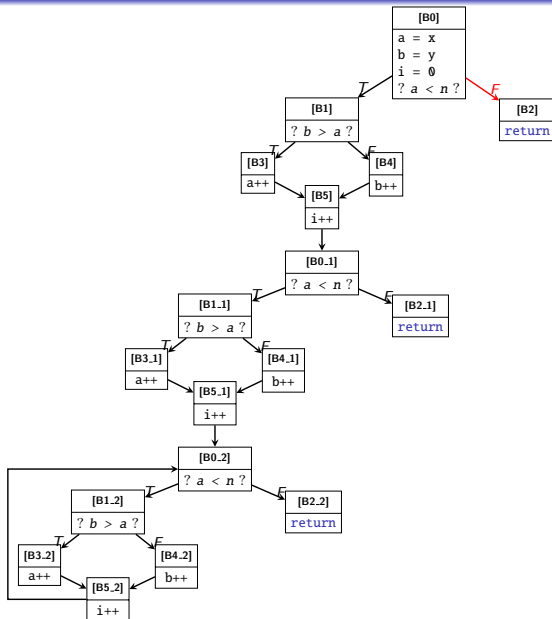
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How about loops?

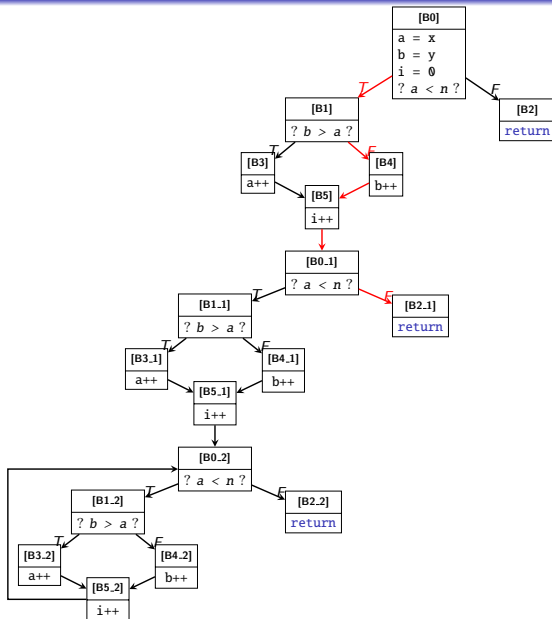
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2 fn sync(
3   x: u64, y: u64, n: u64
4 ) -> (u64, u64, u64) {
5   let a = x, b = y, i = 0;
6   while (a < n) {
7     if (b > a) {
8       a++;
9     } else {
10      b++;
11    }
12    i++;
13  }
14  return (a, b, i);
15 }
```

```
1 // core application logic
2 pub fn main() {
3   let (x, y, n) = input();
4   let (a, b, i) = sync(x, y, n);
5   assert!(i == 0 || i < 2*n);
6   
7 }
```

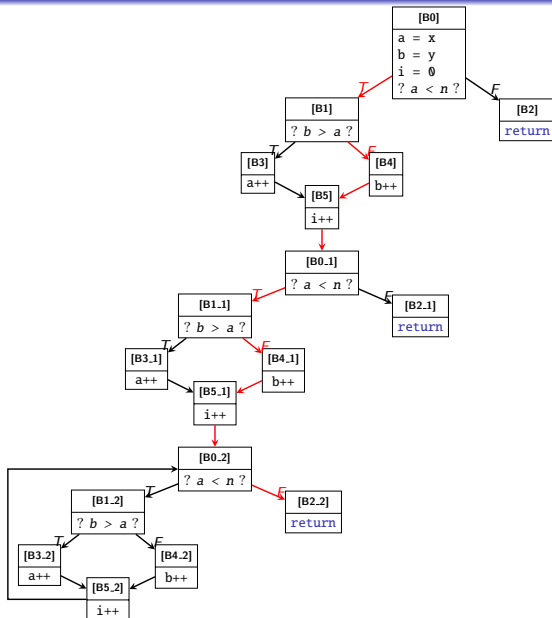
Conventional symbolic execution



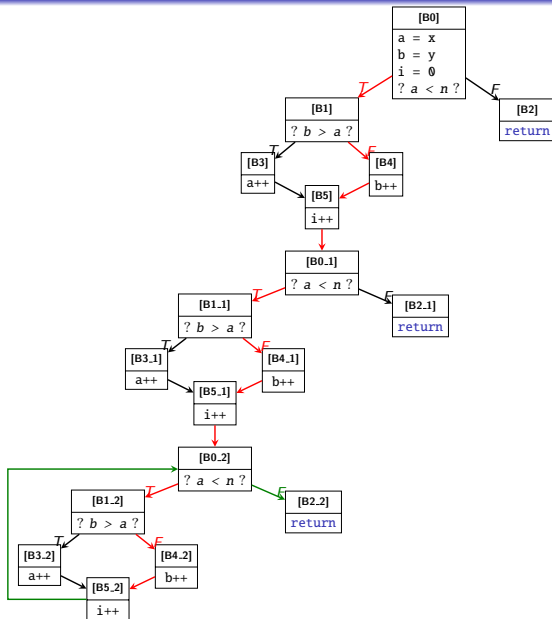
Conventional symbolic execution



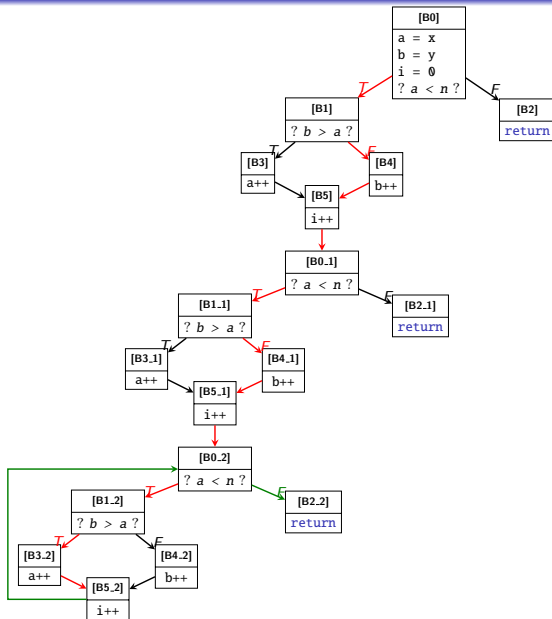
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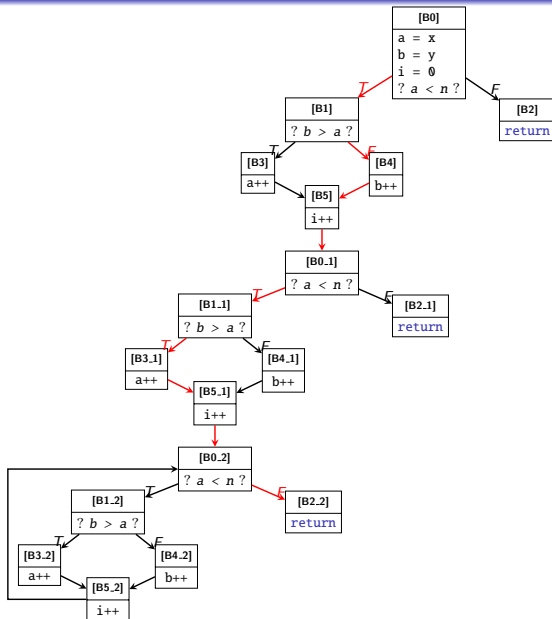
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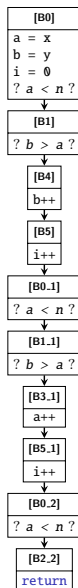
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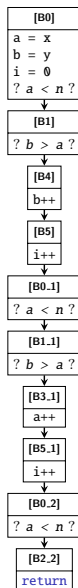
Encoding the path conditions



Find x, y, n such that

- $x < n$ (from [B0])
- $y \leq x$ (from [B1])
- $x < n$ (from [B0.1])
- $y + 1 > x$ (from [B1.1])
- $x + 1 \geq n$ (from [B0.2])
- $n \neq 0 \wedge i \geq 2n$ (from assert!)

Encoding the path conditions



Find x, y, n such that

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- $y \leq x$ (from [B1])
- $x < n$ (from [B0.1])
- $y + 1 > x$ (from [B1.1])
- $x + 1 \geq n$ (from [B0.2])
- $n \neq 0 \wedge i \geq 2n$ (from assert!)

Solving the predicates yield:

$\{ x = 0, y = 0, n = 1 \}$

Symbolic execution for bug finding

```
1 // a library function
2 fn sync(
3   x: u64, y: u64, n: u64
4 ) -> (u64, u64, u64) {
5   let a = x, b = y, i = 0;
6   while (a < n) {
7     if (b > a) {
8       a++;
9     } else {
10      b++;
11    }
12    i++;
13  }
14  return (a, b, i);
15 }
```

- $x=0, y=0, n=1 \rightarrow a=1, b=1, i=2$
- $x=0, y=0, n=2 \rightarrow a=2, b=2, i=4$
-
- $x=0, y=0, n=k \rightarrow a=k, b=k, i=2k$

```
1 // core application logic
2 pub fn main() {
3   let (x, y, n) = input();
4   let (a, b, i) = sync(x, y, n);
5   assert!(i == 0 || i < 2*n);
6   
7 }
```

Path explosion in symbolic execution

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```

Q: What if a bug can only be triggered after exploring k branches?

```
1 // core application logic
2 pub fn main() {
3   let (x, y, n) = input();
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5   assert!(n-a-b+i != 42);
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Path explosion in symbolic execution

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```

Q: What if a bug can only be triggered after exploring k branches?

In fact, this bug can only be triggered after at least 42 levels of loop unrolling.

- $x=0, y=0, n=42 \rightarrow a=42, b=42, i=84$
- $x=9, y=5, n=56 \rightarrow a=56, b=56, i=98$

```
1 // core application logic
2 pub fn main() {
3   let (x, y, n) = input();
4   let (a, b, i) = sync(x, y, n);
5   assert!(n-a-b+i != 42);
6   
7 }
```

In the conventional way of symbolic execution, finding this bug requires an exhaustive search of 2^{42} paths.

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Definition of concolic execution

Background: **concolic**, as the name suggests, is the combination of two English words: *concrete* and *symbolic*, and the order matters!

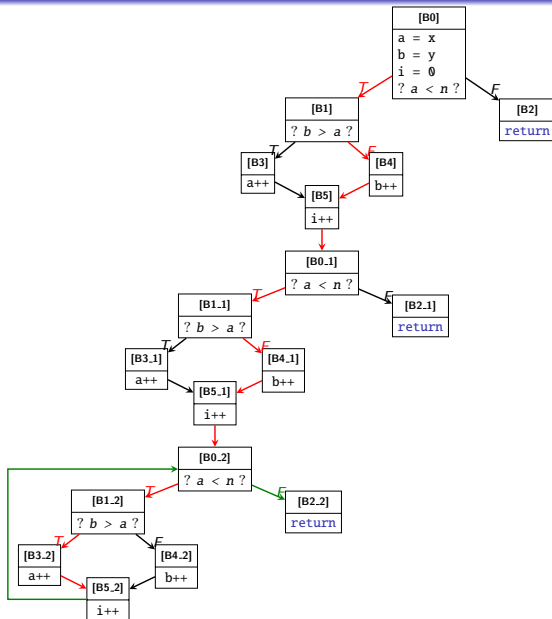
Definition of concolic execution

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The basic idea of **concolic** execution is:

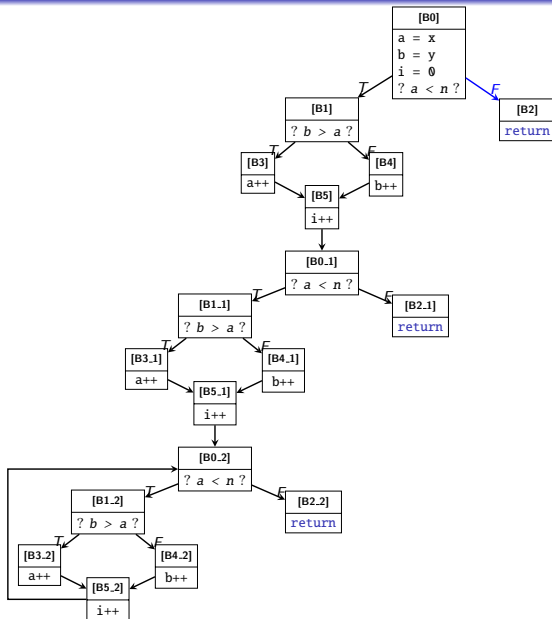
- 1 Execute a test case **concretely**
- 2 For each branch encountered in the test case, find another test case that toggles this branch **symbolically**.

Concolic execution with the running example



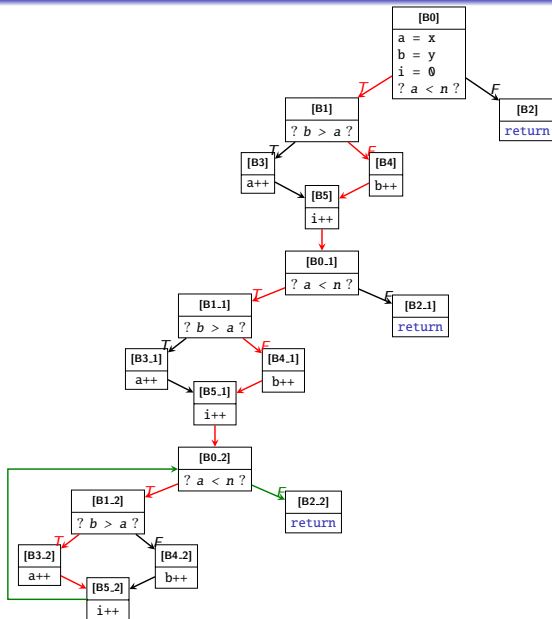
$\{x=1, y=0, n=2\}$

Concolic execution with the running example



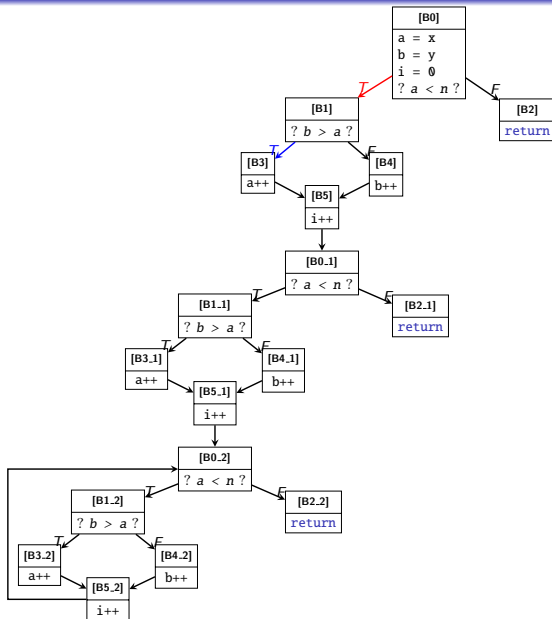
$\{x=9, y=2, n=6\}$

Concolic execution with the running example



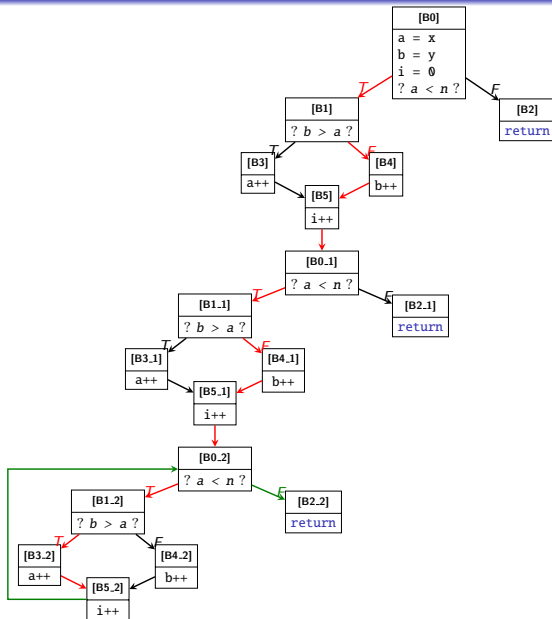
$\{x=1, y=0, n=2\}$

Concolic execution with the running example



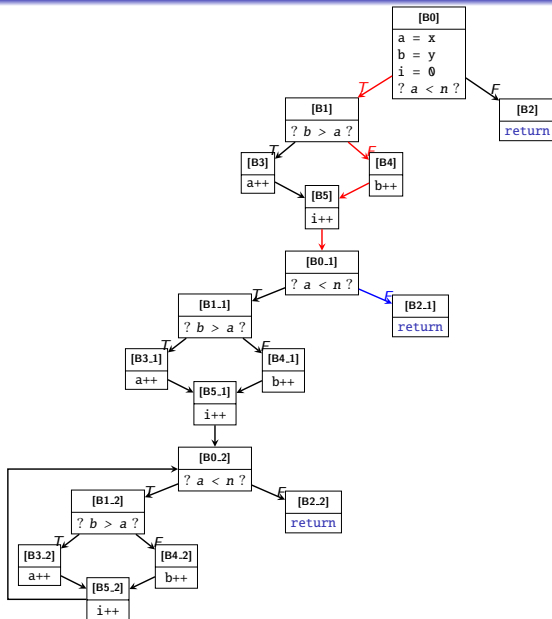
$\{x=3, y=4, n=5\}$

Concolic execution with the running example



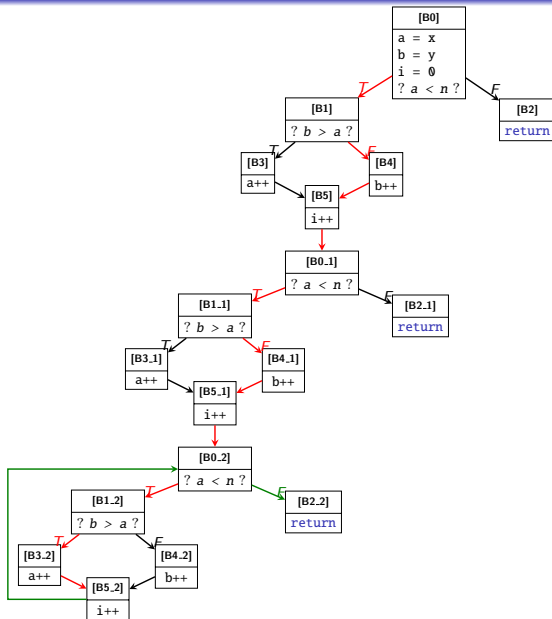
$\{x=1, y=0, n=2\}$

Concolic execution with the running example



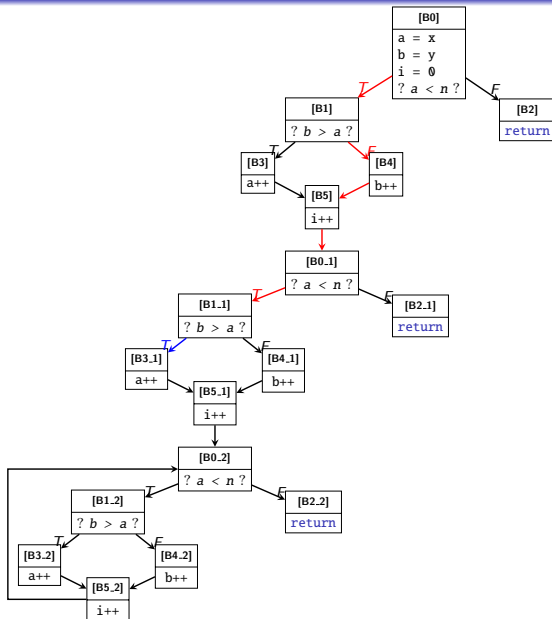
⟨infeasible⟩

Concolic execution with the running example



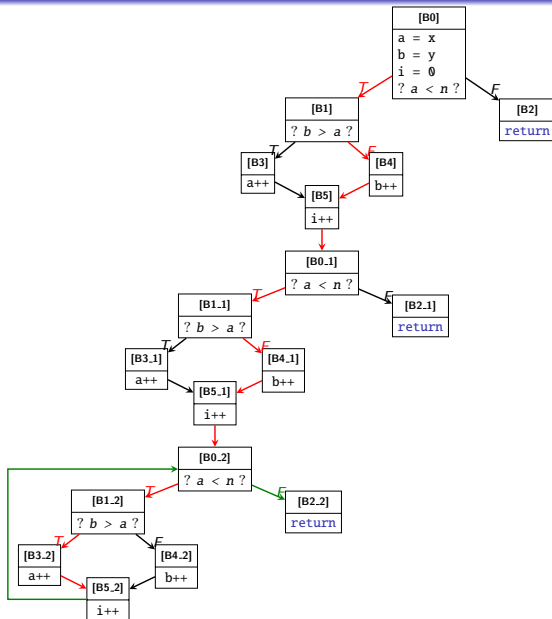
$\{x=1, y=0, n=2\}$

Concolic execution with the running example



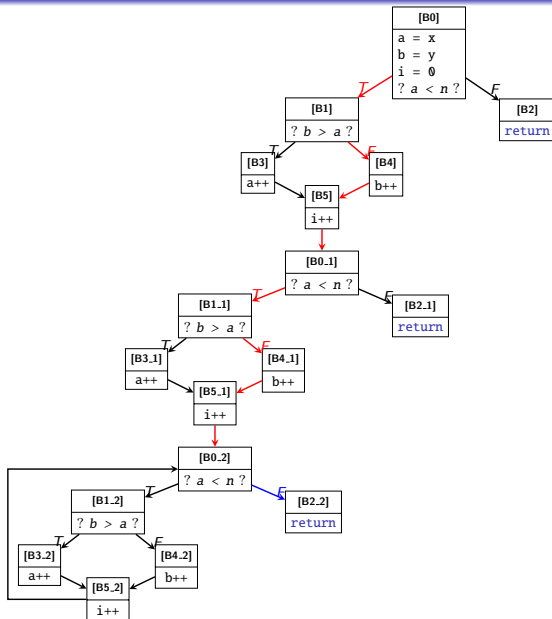
$\{x=5, y=5, n=8\}$

Concolic execution with the running example



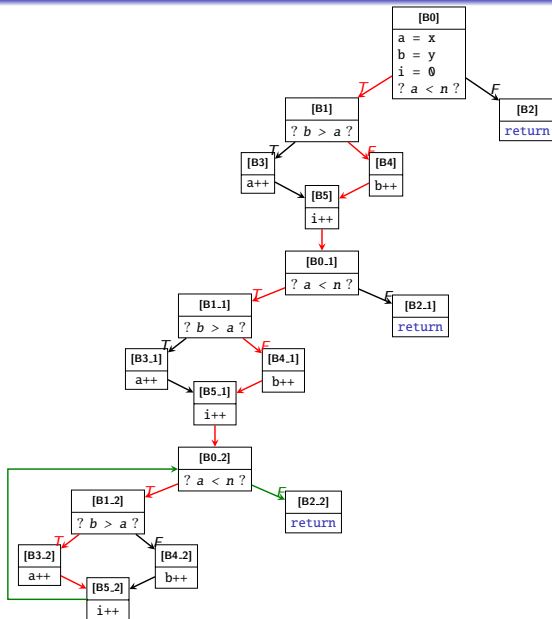
{x=1, y=0, n=2}

Concolic execution with the running example



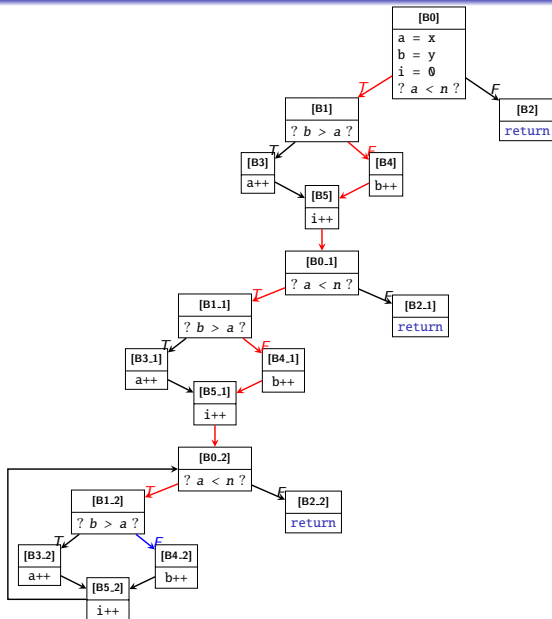
<infeasible>

Concolic execution with the running example



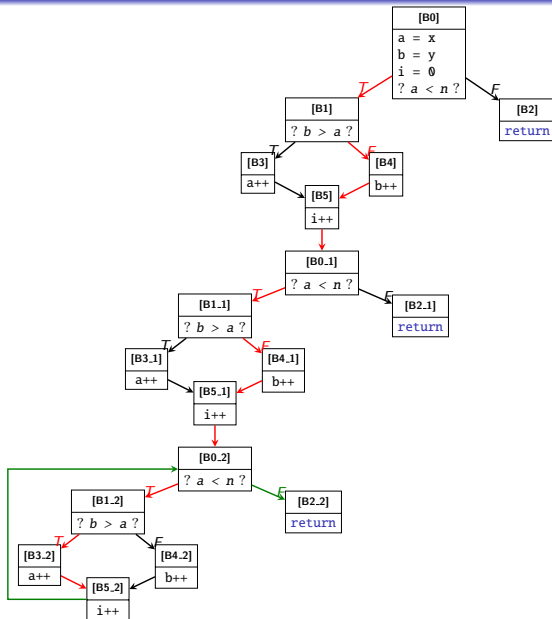
$\{x=1, y=0, n=2\}$

Concolic execution with the running example



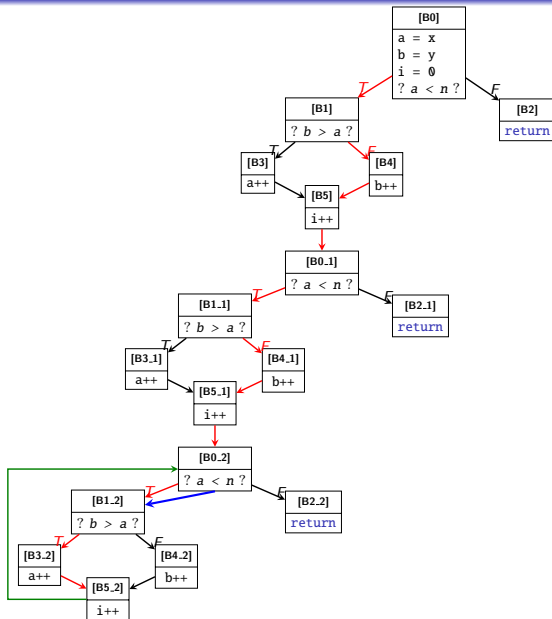
$\{x=7, y=3, n=9\}$

Concolic execution with the running example



$\{x=1, y=0, n=2\}$

Concolic execution with the running example



... endless loop ...

Outline

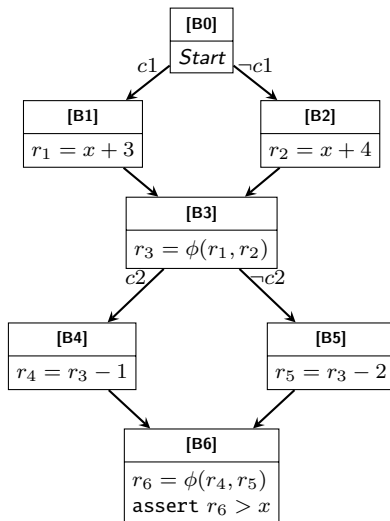
- 1 Introduction
- 2 Conventional symbolic execution
- 3 Symbolic loop unrolling
- 4 Concolic execution and hybrid fuzzing
- 5 Weakest precondition

Weakest precondition calculus

When used in an automated formal verification context, most symbolic executors adopt a **backward** state exploration process, following the **weakest precondition** calculus.

The running example, once again

```
1 fn foo(  
2   c1: bool, c2: bool,  
3   x: u64  
4 ) -> u64 {  
5   let r = if (c1) {  
6     x + 3  
7   } else {  
8     x + 4  
9   };  
10  
11  let r = if (c2) {  
12    r - 1  
13  } else {  
14    r - 2  
15  };  
16  
17  r  
18 }  
19 spec foo {  
20   ensures r > x;  
21 }
```



The passification process

Convert the program into a **dynamic single assignment (DSA)** form.

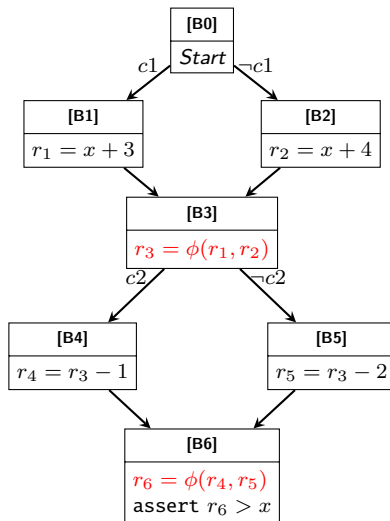
The passification process

Convert the program into a **dynamic single assignment (DSA)** form.

DSA is extremely similar to static single assignment (SSA) with the ϕ -node eagerly uplifted.

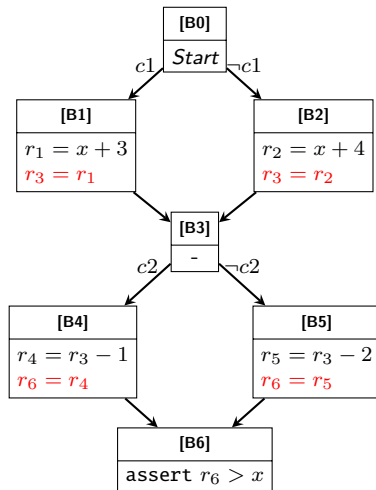
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The walk-up process

Do a **topological sort** on the CFG and traverse backward.

The walk-up process

Do a **topological sort** on the CFG and traverse backward.

This ensures that for each block in the CFG, we visit it *once and only once* (assuming no loops).

The walk-up algorithm

Follow these rules for the intra-block walk-up process:

- $wp(\text{assert } c) = c$
- $wp(\text{assert } c, Q) = c \wedge Q$
- $wp(\text{assign } e, Q) = e \implies Q$
- $wp(s_1; s_2, Q) = wp(s_1, wp(s_2, Q))$

The walk-up algorithm

Follow these rules for the intra-block walk-up process:

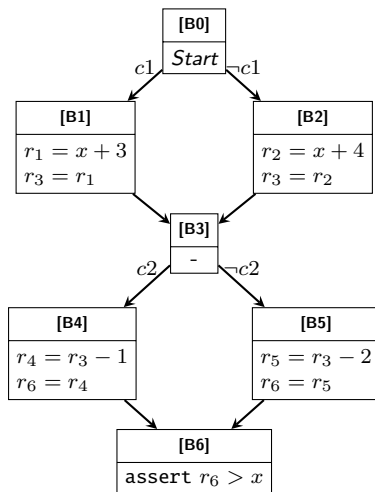
- $wp(\text{assert } c) = c$
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- $wp(\text{assign } e, Q) = e \implies Q$
- $wp(s_1; s_2, Q) = wp(s_1, wp(s_2, Q))$

The rule for inter-block walk-up is:

$$A \leftarrow wp(s_1; s_2; \dots; s_n, \bigwedge_{B \in \text{Succ}(A)} B)$$

The walk-up process with an example

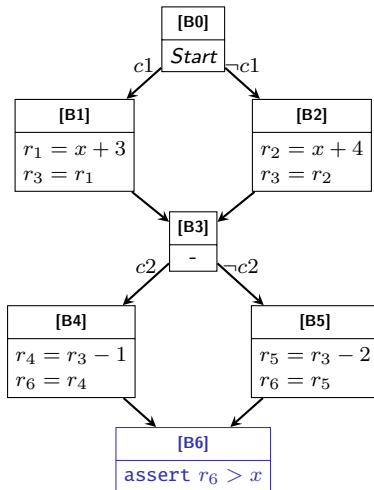
Vars: $c1, c2, x, r_{1-6}, B_{0-6}$



The walk-up process with an example

Vars: $c1, c2, x, r_{1-6}, B_{0-6}$

$B_6 \leftarrow r_6 > x$

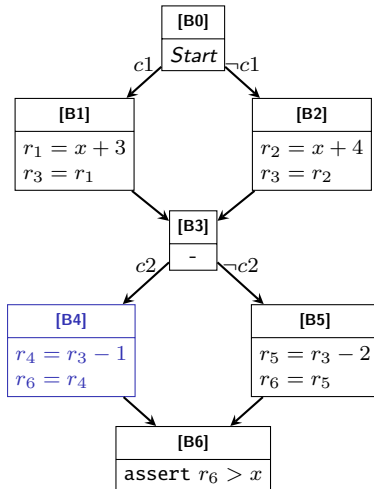


The walk-up process with an example

Vars: $c1, c2, x, r_{1-6}, B_{0-6}$

$B_6 \leftarrow r_6 > x$

$B_4 \leftarrow (c2) \Rightarrow ($
 $(r_4 = r_3 - 1) \Rightarrow ($
 $(r_6 = r_4) \Rightarrow B_6))$



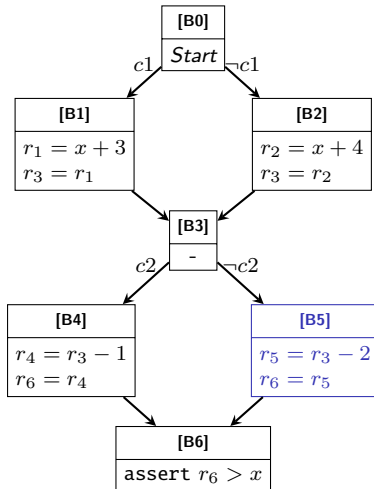
The walk-up process with an example

Vars: $c1, c2, x, r_{1-6}, B_{0-6}$

$B_6 \leftarrow r_6 > x$

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$B_5 \leftarrow (\neg c2) \Rightarrow ($
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The walk-up process with an example

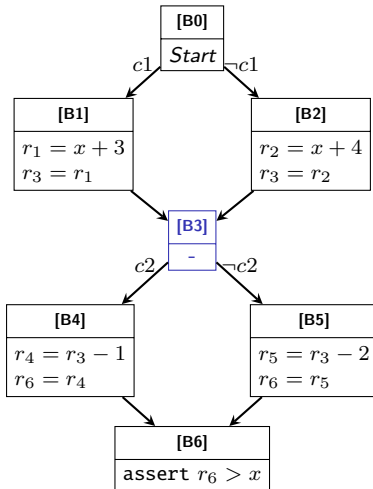
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$B_3 \leftarrow B_4 \wedge B_5$



The walk-up process with an example

Vars: $c1, c2, x, r_{1-6}, B_{0-6}$

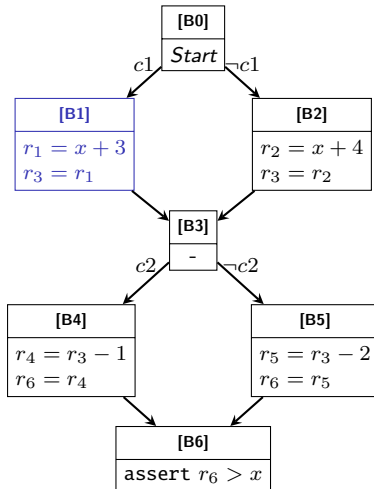
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$B_3 \leftarrow B_4 \wedge B_5$

$B_1 \leftarrow (c1) \Rightarrow ($
 $\quad (r_1 = x + 3) \Rightarrow ($
 $\quad \quad (r_3 = r_1) \Rightarrow B_3))$



The walk-up process with an example

Vars: $c1, c2, x, r_1-6, B_0-6$

$B_6 \leftarrow r_6 > x$

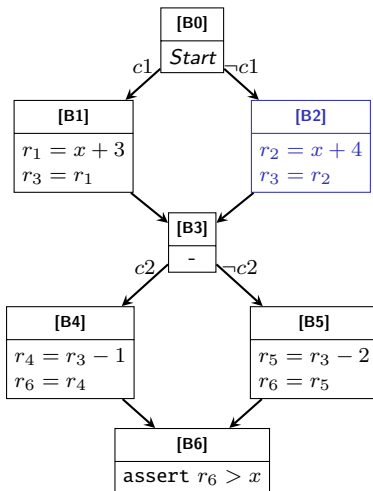
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 $\quad (r_1 = x + 3) \Rightarrow ($
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 $\quad (r_2 = x + 4) \Rightarrow ($
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The walk-up process with an example

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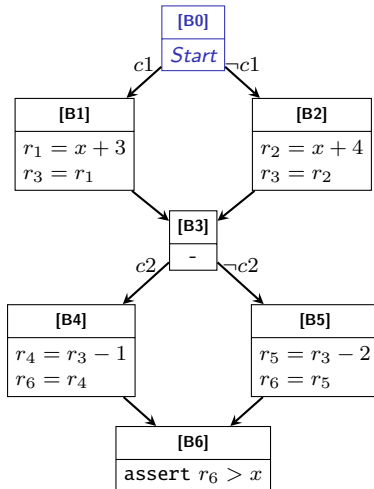
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$B_0 \leftarrow B_1 \wedge B_2$



Proving procedure

Prove that

$\forall c1, c2, x, r_{1-6}, B_{0-6}$:

$B_6 \leftarrow r_6 > x$

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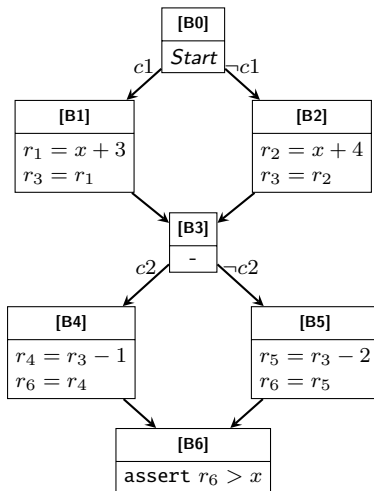
$B_3 \leftarrow B_4 \wedge B_5$

$B_1 \leftarrow (c1) \Rightarrow ($
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 $(r_2 = x + 4) \Rightarrow ($
 $(r_3 = r_2) \Rightarrow B_3))$

$B_0 \leftarrow B_1 \wedge B_2$

$B_0 = \text{True}$



Comparison of forward and backward symbolic execution

Prove that $\forall c1, c2, x, r_{1-6}$:

$$((c1 \wedge c2) \wedge (\\ (r_1 = x + 3) \\ (r_3 = r_1) \\ (r_4 = r_3 - 1) \\ (r_6 = r_4) \\)) \Rightarrow (r_6 > x)$$

However, need to repeat this process multiple (worst case exponential) times.

Prove that

$\forall c1, c2, x, r_{1-6}, B_{0-6}$:

$$\begin{aligned} B_6 &\leftarrow r_6 > x \\ B_4 &\leftarrow (c2) \Rightarrow (\\ &\quad (r_4 = r_3 - 1) \Rightarrow (\\ &\quad \quad (r_6 = r_4) \Rightarrow B_6)) \\ B_5 &\leftarrow (\neg c2) \Rightarrow (\\ &\quad (r_5 = r_3 - 2) \Rightarrow (\\ &\quad \quad (r_6 = r_5) \Rightarrow B_6)) \\ B_3 &\leftarrow B_4 \wedge B_5 \\ B_1 &\leftarrow (c1) \Rightarrow (\\ &\quad (r_1 = x + 3) \Rightarrow (\\ &\quad \quad (r_3 = r_1) \Rightarrow B_3)) \\ B_2 &\leftarrow (\neg c1) \Rightarrow (\\ &\quad (r_2 = x + 4) \Rightarrow (\\ &\quad \quad (r_3 = r_2) \Rightarrow B_3)) \\ B_0 &\leftarrow B_1 \wedge B_2 \end{aligned}$$

$$B_0 = \text{True}$$

⟨ End ⟩